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KINETIC THEORY OF RAPIDLY VARIABLE PROCESSES IN A PLASMA

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INTRODUCTION

Boltzmann's kinetic equation is unsuitable in conditions in which, firstly, the characteristic dimensions of the inhomogeneities are found to be comparable with the characteristic impact parameters of particles taking part in collisions, and secondly, in which during the characteristic time of change of distribution of the particles the collision begun does not have time to be completed[1]. The generalization of Boltzmann's equations to the case of distinctly inhomogeneous problems was given in the book by Chapman and Cowling[2] for the case of a gas of impenetrable spheres and in the book by Bogoliubov[3] for the construction of generalized Boltzmann's equations for the arbitrary law of interaction of the particles of a gas.

The exposition below is devoted to the obtaining of a kinetic equation for a gas of particles with weak interaction suitable for the description of rapidly variable processes and to certain of its applications.

An example of such a gas is a plasma in which the particles interact in accordance with the Coulomb force law, with a considerable contribution to the scattering of the particles being made by long-range collisions, for which interaction may be considered weak. As a result of the slowness of the decrease of Coulomb forces the times during which long-range collisions may occur lie within a broad interval from

$$\tau_{min} \sim e^2 m^{\frac{1}{2}} (\varkappa T)^{-\frac{3}{2}} \sim 10^{-8} T^{-\frac{3}{2}}$$

to

$$\tau_{max} \sim \sqrt{\frac{m}{4\pi e^2 n_o}} = \omega_o^{-1} \sim 10^{-5} n_o^{-\frac{1}{2}} .$$

Here T is the temperature and n_o is the density of the electrons in a unit of volume (ω_o is the Langmuir frequency).

In conditions in which the characteristic time of the process τ_{pr} satisfies the inequalities

$$\tau_{min} << \tau_{pr} << \tau_{max}$$

firstly, there is an extensive region ($\tau_{min} << \tau_{process}$), in which not only the interaction may be considered weak, but the interactions may be considered without regard to a change of distribution in time, and, secondly, there is also an extensive region ($\tau_{process} << \tau_{max}$) in which the interactions change substantially. Actually in the latter region, due to the rapid change of the distribution of the particles, the collisions are suppressed and $\tau_{process}$ plays the role of the maximum time of collision. This fact is well known from the theory of the absorption of radio waves in interplanetary gas[4].

Section 1 of the present communication is devoted to the obtaining of a nonrelativistic kinetic equation for a plasma, and § 2, correspondingly, to the obtaining of a relativistic equation, which are suitable for the investigation of rapidly variable processes. Finally, § 3 examines the application of the equation obtained in § 1 to the theory of the high-frequency dielectric permeability of a plasma.

§ 1. Kinetic Equation for Rapidly Variable Processes in a Nonrelativistic Plasma.

Let us investigate a nonrelativistic plasma in a constant and homogeneous magnetic field $\vec{B}$ and in an electrical field $\vec{E}$, likewise homogeneous, but, generally speaking, varying in time. The fact that the plasma is nonrelativistic makes it possible for us to confine ourselves to taking into account only the Coulomb interaction of the particles of the plasma ($U_{\alpha\beta}(\vec{r}) = e_\alpha e_\beta / r$). In the assumption that the interaction of the particles of the plasma is weak we may write the following equations for the distribution function of the particles of the α-type f_α and for the correlation function $g_{\alpha\beta}$ (cf. ref. 3):

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}_\alpha} + e_\alpha(\vec{E} + \frac{1}{c}[\vec{v}_\alpha \vec{B}]) \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} =$$

$$\sum_\beta \frac{N_\beta}{V} \int d\vec{p}_\beta d\vec{r}_\beta \frac{\partial U_{\alpha\beta}(|\vec{r}_\alpha - \vec{r}_\beta|)}{\partial \vec{r}_\alpha} \frac{\partial g_{\alpha\beta}(\vec{p}_\alpha, \vec{p}_\beta, \vec{r}_\alpha, \vec{r}_\beta, t)}{\partial \vec{p}_\alpha} \qquad (1.1)$$

$$\left\{\frac{\partial}{\partial t} + \vec{v}_\alpha \frac{\partial}{\partial \vec{r}_\alpha} + \vec{v}_\beta \frac{\partial}{\partial \vec{r}_\beta} + e_\alpha(\vec{E} + \frac{1}{c}[\vec{v}_\alpha \vec{B}]) \frac{\partial}{\partial \vec{p}_\alpha} +\right.$$

$$\left. + e_\beta(\vec{E} + \frac{1}{c}[\vec{v}_\beta \vec{B}]) \frac{\partial}{\partial \vec{p}_\beta}\right\} g_{\alpha\beta} = \frac{\partial U_{\alpha\beta}(|\vec{r}_\alpha - \vec{r}_\beta|)}{\partial \vec{r}_\alpha} \left(\frac{\partial}{\partial \vec{p}_\alpha} - \frac{\partial}{\partial \vec{p}_\beta}\right) f_\alpha f_\beta \qquad (1.2)$$

The solution of equation (1.2) may be written in the form:

$$g_{\alpha\beta}(\vec{p}_\alpha, \vec{p}_\beta, \vec{r}_\alpha, \vec{r}_\beta, t) = g_{\alpha\beta}(\vec{P}_\alpha[t_o, t, \vec{p}_\alpha], \vec{P}_\beta[t_o, t, \vec{p}_\beta],$$

$$\vec{R}_\alpha[t_o, t, \vec{p}_\alpha, \vec{r}_\alpha], \vec{R}_\beta[t_o, t, \vec{p}_\beta, \vec{r}_\beta], t_o) + \int_{t_o}^{t} dt'$$

$$\left\{\frac{\partial}{\partial \vec{r}_\alpha} U_{\alpha\beta}(|\vec{R}_\alpha[t', t, \vec{p}_\alpha, \vec{r}_\alpha] - \vec{R}_\beta[t', t, \vec{p}_\beta, \vec{r}_\beta]|)\right\}\left\{\frac{\partial}{\partial \vec{P}_\alpha[t', t, \vec{p}_\alpha]} -\right.$$

$$\left. - \frac{\partial}{\partial \vec{P}_\beta[t', t, \vec{p}_\beta]}\right\} f_\alpha(\vec{P}_\alpha[t, t', \vec{p}_\alpha], \vec{R}_\alpha[t, t', \vec{p}_\alpha, \vec{r}_\alpha], t')$$

$$f_\beta(\vec{P}_\beta \; \vec{R}_\beta \; t'), \qquad (1.3)$$

where

$$\vec{r}_\alpha[t',t,\vec{p}_\alpha] = \vec{B}\,\frac{(\vec{B}\vec{p}_\alpha)}{B^2} - \sin\Omega_\alpha(t'-t)\,\frac{[\vec{B}\vec{p}_\alpha]}{B} -$$

$$- \cos\Omega_\alpha(t'-t)\,\frac{\left[\vec{B}[\vec{B}\vec{p}_\alpha]\right]}{B^2} + e_\alpha\int\limits_t^{t'} dt''\left\{\vec{B}\,\frac{\vec{B}\vec{E}(t'')}{B^2} -\right.$$

$$\left.- \frac{[\vec{B}\vec{E}(t'')]}{B}\sin\Omega_\alpha(t'-t'') - \cos\Omega_\alpha(t'-t'')\,\frac{\left[\vec{B}[\vec{B}\vec{E}(t'')]\right]}{B^2}\right\} \quad (1.4)$$

$$\vec{R}_\alpha[t',t,\vec{p}_\alpha,\vec{r}_\alpha] = \vec{r}_\alpha + \vec{B}\,\frac{(\vec{B}\vec{v}_\alpha)}{B^2}(t'-t) - \frac{1-\cos\Omega_\alpha(t'-t)}{\Omega_\alpha}\,\frac{[\vec{B}\vec{v}_\alpha]}{B}$$

$$- \frac{\sin\Omega_\alpha(t'-t)}{\Omega_\alpha}\,\frac{\left[\vec{B}[\vec{B}\vec{v}_\alpha]\right]}{B^2} + \frac{e_\alpha}{m_\alpha}\int\limits_t^{t'} dt''\int\limits_t^{t''} dt'''$$

$$\left\{\vec{B}\,\frac{\left(\vec{B}\vec{E}(t''')\right)}{B^2} - \sin\Omega_\alpha(t''-t''')\,\frac{[\vec{B}\vec{E}(t''')]}{B} -\right.$$

$$\left.- \cos\Omega_\alpha(t''-t''') - \frac{\left[\vec{B}[\vec{B}\vec{E}(t''')]\right]}{B^2}\right\} \quad (1.5)$$

The correlation function (1.3), on substitution in equation (1.1) yields a kinetic equation suitable for the investigation of rapidly variable processes. For steady-state processes in formula (1.3) we may omit $g_{\alpha\beta}(t_o)$ and assume $t_o = -\infty$. Then the kinetic equation may be written in the following form

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha\,\frac{\partial f_\alpha}{\partial \vec{r}_\alpha} + e_\alpha(\vec{E} + \frac{1}{c}[\vec{v}_\alpha\vec{B}])\,\frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = \sum_\beta \frac{N_\beta}{V}\int d\vec{p}_\beta d\vec{r}_\beta \frac{\partial U_{\alpha\beta}}{\partial r^i_\alpha}(\vec{r}_\alpha - \vec{r}_\beta)$$

$$\frac{\partial}{\partial p^i_\alpha}\int\limits_{-\infty}^{t} dt'\left\{\frac{\partial}{\partial r^j_\alpha}\,U_{\alpha\beta}(|\vec{R}_\alpha[t',t,\vec{p}_\alpha,\vec{r}_\alpha] - \vec{R}_\beta[t',t,\vec{p}_\beta,\vec{r}_\beta]|)\right\}$$

$$\left(\frac{\partial}{\partial \vec{P}^{j}_{\alpha}[t',t,\vec{p}_{\alpha}]} - \frac{\partial}{\partial \vec{P}^{j}_{\beta}[t',t,\vec{p}_{\beta}]}\right) f_{\alpha}(\vec{P}_{\alpha},\vec{R}_{\alpha},t') f_{\beta}(\vec{P}_{\beta},\vec{R}_{\beta},t') \qquad (1.6)$$

Inasmuch as equation (1.6) is obtained in the assumption that the electromagnetic field is spatially homogeneous, then, strictly speaking, it is applicable only for spatially homogeneous distributions. Actually, the region of applicability of this equation is broader. In particular, the collision integral of equation (1.6) may be utilized for the description of dissipation processes in the plasma if the self-consistent field arising supplementally to the constant electric and magnetic fields has a negligibly weak effect on the collisions of the particles.

It should be said that equation (1.6) is unsuitable in conditions when it is impossible to utilize the theory of perturbations. This is the case in particular for sufficiently small collision parameters, when the collision integral (1.6) is logarithmically divergent. Keeping in view the inapplicability of our treatment for small collision parameters, in integration with respect to the impact parameters integration should be broken off at p_{min}. Here the suspension of perturbation theory begins at $p_{min} \sim e^2/\varkappa T$. On the other hand, the minimum value of the impact parameter may be determined by the inapplicability of classical mechanics. In this case $p_{min} \sim \hbar/mv$.

The collision integral (1.6) does not take into account the screening of the Coulomb interaction. Therefore in integration in the right member of this equation the integral for large impact parameters should be broken off at $p_{max} = r_D$, where r_D is the Debye radius. In the case of slowly varying processes in the absence of a magnetic field the right member of equation (1.6) is the collision integral of Landau[5]. For rapidly variable processes, whose characteristic frequency of variation is greater than the Langmuir frequency of the electrons, breaking off on the part of the large impact parameters occurs automatically. It is in these conditions that our kinetic equation differs essentially from that proceeding from the usual scheme of collisions of Boltzmann.

§ 2. Relativistic Kinetic Equation for Rapidly Variable Processes

Above the kinetic equation was obtained for nonrelativistic particles, describing processes varying perceptibly during the time of collision. Here the corresponding kinetic equation will be found for the case of relativistic particles. Let us note that in the relativistic case, for example, in the propagation of an electromagnetic wave with a period substantially smaller than the characteristic time of collision there simultaneously arises a spatial inhomogeneity of distribution substantially smaller in dimensions than the characteristic impact parameter of the collision. It is for this reason that the collision integral obtained below is nonlocal both with respect to time and with respect to coordinates.

Let us note that the relativistic collision integral for distributions slowly varying in time and sharply varying in space was obtained in the study of Klimontovich[6]. For smooth distributions the corresponding collision integral was obtained by Beliaev and Budker[7] (see also ref. 6). In the latter case the relativistic collision integral contains a large logarithm arising from integration with respect to the impact parameters $L = \ln(p_{max}/p_{min})$. Here p_{max} is the Debye radius of screening (see also ref. 8). Therefore the difference between the collision integral obtained below and the usual collision integral[7] will be manifested under conditions in which, on the one hand, the characteristic dimension of the inhomogeneity is not large in comparison with the Debye radius, and on the other hand, when the characteristic time of variation of the process is smaller than p_{max}/v --the time of collision with the impact parameter p_{max}.

In describing a system of charged particles, strictly speaking, it is necessary to consider not only the particles, but also the electromagnetic field. However, in order to obtain a kinetic equation taking into account the collisions of the particles of a gas with weak interaction one may proceed more simply. Here it is expedient to single out the self-consistent field which, as usual, is described by Maxwell equations with the current and charge density determined by the distribution function of the particles. In order to find the force correlation resulting from collisions let us utilize the expression for the force acting on a particle α due to a particle β moving uniformly.

$$\vec{F}_{\alpha,\beta}(\vec{r}_\alpha - \vec{r}_\beta, \vec{v}_\alpha, \vec{v}_\beta) = 4\pi e_\alpha e_\beta i \int \frac{d\vec{k}}{(2\pi)^3}$$

$$\frac{\exp(i\vec{k}, \vec{r}_\alpha - \vec{r}_\beta)}{K^2 - (\vec{K}\vec{v}_\beta/c)^2} \left\{ -\vec{K}\left(1 - \frac{(\vec{v}_\alpha \vec{v}_\beta)}{c^2}\right) + \vec{v}_\beta \frac{(\vec{K}, \vec{v}_\beta - \vec{v}_\alpha)}{c^2} \right\} \tag{2.1}$$

Then for the distribution function f_α and the pair correlation function $g_{\alpha\beta}$ the following equations may be written (cf. ref. 13):

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}_\alpha} + e_\alpha \left\{ \vec{E}(\vec{r}_\alpha) + \frac{1}{c} [\vec{v}_\alpha \vec{B}(\vec{r}_\alpha)] \right\} \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} =$$

$$= \sum_\beta \frac{N_\beta}{V} \int d\vec{p}_\beta d\vec{r}_\beta \left(\frac{\partial g_{\alpha\beta}}{\partial \vec{p}_\alpha} \vec{F}_{\alpha,\beta} \right) \tag{2.2}$$

$$\left\{ \frac{\partial}{\partial t} + \vec{v}_\alpha \frac{\partial}{\partial \vec{r}_\alpha} + \vec{v}_\beta \frac{\partial}{\partial \vec{r}_\beta} + e_\alpha \left(\vec{E}(\vec{r}_\alpha) + \frac{1}{c}[\vec{v}_\alpha \vec{B}(\vec{r}_\alpha)] \right) \frac{\partial}{\partial \vec{p}_\alpha} + \right.$$

$$\left. + e_\beta \left(\vec{E}(\vec{r}_\beta) + \frac{1}{c}[\vec{v}_\beta \vec{B}(\vec{r}_\beta)] \right) \frac{\partial}{\partial \vec{p}_\beta} + F_{\alpha,\beta} \frac{\partial}{\partial \vec{p}_\alpha} + F_{\beta,\alpha} \frac{\partial}{\partial \vec{p}_\beta} \right\} g_{\alpha\beta} +$$

$$+ \vec{F}_{\alpha,\beta} f_\beta \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} + F_{\beta,\alpha} f_\alpha \frac{\partial f_\beta}{\partial \vec{p}_\beta} + \sum_\gamma{}' \frac{N_\gamma}{V} \int d\vec{p}_\gamma d\vec{r}_\gamma \left\{ \vec{F}_{\alpha,\gamma} \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} g_{\beta\gamma} + \right.$$

$$\left. + \vec{F}_{\beta,\gamma} \frac{\partial f_\beta}{\partial \vec{p}_\beta} g_{\alpha\gamma} + \vec{F}_{\alpha,\gamma} \frac{\partial g_{\alpha\beta\gamma}}{\partial \vec{p}_\alpha} + \vec{F}_{\beta,\gamma} \frac{\partial g_{\alpha\beta\gamma}}{\partial \vec{p}_\beta} \right\} = 0 \qquad (2.3)$$

In order to obtain the kinetic equation it is necessary for us to find the expression of the binary correlation function determined by equation (2.3). In the assumption of weakness of interaction the latter may be substantially simplified. Firstly, from a comparison of the different terms of this equation it is clear that the binary correlation function will be of an order of (U/T) in comparison with $f_\alpha f_\beta$, where U/T is the ratio of the energy of interaction to the kinetic energy. Therefore in equation (2.3) we may disregard the terms

$$\vec{F}_{\alpha,\beta} \frac{\partial g_{\alpha\beta}}{\partial \vec{p}_\alpha} + \vec{F}_{\beta,\alpha} \frac{\partial g_{\alpha\beta}}{\partial \vec{p}_\beta}$$

Secondly, writing the corresponding equation for the ternary correlation function it is easy to see that it will also be of an order U/T in comparison with $f_\alpha g_{\beta\gamma}$. This makes it possible in equation (2.3) to disregard the terms

$$\sum_\gamma{}' \frac{N_\gamma}{V} \int d\vec{p}_\gamma d\vec{r}_\gamma \left\{ \vec{F}_{\alpha,\gamma} \frac{\partial g_{\alpha\beta\gamma}}{\partial \vec{p}_\alpha} + \vec{F}_{\beta,\alpha} \frac{\partial g_{\alpha\beta\gamma}}{\partial \vec{p}_\beta} \right\}$$

in comparison with the analogous terms containing the binary correlation function. Thirdly, our assumption concerning the possibility of describing scattering by means of the force (1) may obtain only in conditions in which the fields $\vec{E}$ and $\vec{B}$ are so small that they have no effect on the trajectory of the particle during collision. This assumption corresponds to the possibility of disregarding the fields $\vec{E}$ and $\vec{B}$ in equation (2.3). As a result, for the correlation function we may write the following equation:

$$\left(\frac{\partial}{\partial t} + \vec{v}_\alpha \frac{\partial}{\partial \vec{r}_\alpha} + \vec{v}_\beta \frac{\partial}{\partial \vec{r}_\beta}\right) g_{\alpha\beta} + \vec{F}_{\alpha,\beta} f_\beta \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} + \vec{F}_{\beta,\alpha} f_\alpha \frac{\partial f_\beta}{\partial \vec{p}_\beta} =$$

$$= - \sum_\gamma \frac{N_\gamma}{V} \int d\vec{p}_\gamma d\vec{r}_\gamma \left\{ \vec{F}_{\alpha,\gamma} \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} g_{\beta\gamma} + \vec{F}_{\beta,\gamma} \frac{\partial f_\beta}{\partial \vec{p}_\beta} g_{\alpha\gamma} \right\} \qquad (2.4)$$

The right member of equation (2.4) is essential for screening of the Coulomb interaction in the case of slowly varying processes. If, however, the distribution of the particles varies substantially during a time that is small in comparison with the characteristic time corresponding to the screening of the Coulomb interaction, then the right member of equation (2.4) need not be taken into account. In this case the solution of equation (2.4) may be written in the form

$$g_{\alpha\beta}(\vec{r}_\alpha, \vec{r}_\beta, \vec{p}_\alpha, \vec{p}_\beta, t) = g^{(0)}_{\alpha\beta}\ (\vec{r}_\alpha - \vec{v}_\alpha(t-t_o), \vec{r}_\beta - \vec{v}_\beta(t-t_o), \vec{p}_\alpha, \vec{p}_\beta, t_o)$$

$$- \int_{t_o}^{t} dt' \Big\{ f_\alpha(\vec{r}_\alpha + \vec{v}_\alpha(t'-t), \vec{p}_\alpha, t')\ \vec{F}_{\beta,\alpha}(\vec{r}_\beta - \vec{r}_\alpha + (\vec{v}_\beta - \vec{v}_\alpha)(t'-t); \vec{v}_\beta, \vec{v}_\alpha)$$

$$\left(\frac{\partial}{\partial \vec{p}_\beta} - \frac{\partial v^i_\beta}{\partial \vec{p}_\beta}(t'-t)\frac{\partial}{\partial r^i_\beta}\right) f_\beta(\vec{r}_\beta+\vec{v}_\beta(t'-t), \vec{p}_\beta, t') +$$

$$+ f_\beta(\vec{r}_\beta+\vec{v}_\beta(t'-t), \vec{p}_\beta, t')\ \vec{F}_{\alpha,\beta}(\vec{r}_\alpha-\vec{r}_\beta+(\vec{v}_\alpha-\vec{v}_\beta)(t'-t), \vec{v}_\alpha, \vec{v}_\beta)$$

$$\left(\frac{\partial}{\partial \vec{p}_\alpha} - \frac{\partial v^i_\alpha}{\partial \vec{p}_\alpha}(t'-t)\frac{\partial}{\partial r^i_\alpha}\right) f_\alpha(\vec{r}_\alpha+\vec{v}_\alpha(t'-t); \vec{p}_\alpha, t')\Big\} \qquad (2.5)$$

Formula (2.5) now makes it possible to write eqution (2.2) as the equation for the distribution function f_α. However this equation will also contain an initial correlation function, which will cause it to differ substantially from the usual kinetic equation.

If we investigate steady-state processes taking place in a time considerably exceeding, for example, the characteristic time of collision, then t_o may be assumed equal to $-\infty$, and the initial correlation function $g^{(0)}_{\alpha\beta}$ may be disregarded. Then:

$$g_{\alpha\beta}(r_\alpha, r_\beta, p_\alpha, p_\beta, t) = -\int_{-\infty}^{0} d\tau \Big\{ F_{\beta,\alpha}(r_\beta - r_\alpha + (v_\beta - v_\alpha)\tau; v_\beta; v_\alpha)$$

$$f_\alpha(r_\alpha+v_\alpha\tau, p_\alpha, t+\tau)\left(\frac{\partial}{\partial \vec{p}_\beta} - \frac{\partial v^i_\beta}{\partial \vec{p}_\beta}\tau\frac{\partial}{\partial r^i_\beta}\right) f_\beta(r_\beta+v_\beta\tau, p_\beta, t+\tau) +$$

$$+ \vec{F}_{\alpha,\beta}(r_\alpha - r_\beta + (v_\alpha - v_\beta)\tau; v_\alpha; v_\beta)\ f_\beta(r_\beta+v_\beta\tau, p_\beta, t+\tau)$$

$$\left(\frac{\partial}{\partial \vec{p}_\alpha} \quad \frac{\partial v^i_\alpha}{\partial \vec{p}_\alpha}\tau\frac{\partial}{\partial r^i_\alpha}\right) f_\alpha(r_\alpha+v_\alpha\tau, p_\alpha, t+\tau)\Big\} \qquad (2.6)$$

In substituting expression (2.6) in the left part of equation (2.2) the fact should be taken into account that our treatment is inaccurate in conditions in which the particles approach each other to distances at which their energy of interaction becomes no longer small in comparison with their kinetic energy. This inaccuracy is manifested in the appearance of logarithmic divergence for small values of τ. Therefore below in integration with respect to τ let us limit the region of interaction to τ_{min}. After all the foregoing we may write the kinetic equation sought in the form

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial r_\alpha} + e_\alpha \left\{ \vec{E} + \frac{1}{c}[\vec{B}] \right\} \frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = \mathfrak{J}_\alpha \qquad (2.7)$$

where the collision integral $\mathfrak{J}_\alpha$ has the following form:

$$\mathfrak{J}_\alpha = -\sum_\beta \frac{N_\beta}{V} \frac{\partial}{\partial p_\alpha^i} \int dp_\beta dr_\beta F_{\alpha,\beta}(r_\alpha - r_\beta v_\alpha v_\beta) \int_{-\infty}^{0} d\tau$$

$$\Big\{ F^j_{\beta,\alpha}(r_\beta - r_\alpha + (v_\beta - v_\alpha)\tau, v_\beta, v_\alpha)\ f_\alpha(r_\alpha + v_\alpha\tau, p_\alpha t+\tau)$$

$$\left(\frac{\partial}{\partial p_\beta^j} - \frac{\partial v_\beta^\ell}{\partial p_\beta^j} \tau \frac{\partial}{\partial r_\beta^\ell} \right) f_\beta(r_\beta + v_\beta\tau, p_\beta, t+\tau) +$$

$$+ F^j_{\alpha,\beta}(r_\alpha - r_\beta + (v_\beta + v_\alpha)\tau, v_\alpha, v_\beta)\ f_\beta(r_\beta + v_\beta\tau, p_\beta t+\tau)$$

$$\left(\frac{\partial}{\partial p_\alpha^j} - \frac{\partial v_\alpha^\ell}{\partial p_\alpha^j} \tau \frac{\partial}{\partial r_\alpha^\ell} \right) f_\alpha(r_\alpha + v_\alpha\tau, p_\alpha, t+\tau) \Big\} \qquad (2.8)$$

In the relativistic case for processes with weak spatial inhomogeneities, when we may disregard the dependence of the distribution functions in the collision integral on the coordinates, and also for sufficiently slow variation in time the collision integral (2.8) becomes the corresponding expression of the study by Beliaev and Budker[7]. Here the distributions must vary weakly over distances of an order of the Debye radius and during a time of an order of the period of the plasma oscillations.

It is not difficult to proceed somewhat farther and take into account the effect of the self-consistent field on the collision of the particles, considering this effect as a correction effect. For this let us obtain the correction to the correlation function, retaining in equation (2.3) terms with a self-consistent field, without considering that the function accompanying them has the form (2.5). Then for the correction to the correlation function in the case of steady-state processes we obtain:

$$\delta g_{\alpha\beta}(r_\alpha, r_\beta, p_\alpha, p_\beta, t) = - \int_{-\infty}^{t} dt' (e_\alpha \Big\{ E(r_\alpha + v_\alpha(t'-t), t') +$$

$$+ \frac{1}{c} [v_\alpha, B(r_\alpha + v_\alpha(t'-t), t')] \Big\} \Big\{ \frac{\partial}{\partial p_\alpha} - \frac{\partial v_\alpha^i}{\partial p_\alpha} (t'-t) \frac{\partial}{\partial r_\alpha^i} \Big\} +$$

$$+ e_\beta \Big\{ E(r_\beta + v_\beta(t'-t), t') + \frac{1}{c} [v_\beta B(r_\beta + v_\beta(t'-t), t')] \Big\}$$

$$\left\{\frac{\partial}{\partial p_\beta} - \frac{\partial v_\beta^i}{\partial p_\beta}(t'-t)\frac{\partial}{\partial r_\beta^i}\right\} g_{\alpha\beta}(r_\alpha + v_\alpha(t'-t), r_\beta + v_\beta(t'-t),$$

$$p_\alpha,\ p_\beta,\ t') \qquad (2.9)$$

Formula (2.9) makes it possible to obtain the following correction term for the right member of equation (2.7):

$$\delta\pi_\alpha = \sum_\beta \frac{N_\beta}{V}\frac{\partial}{\partial p_\alpha^i}\int dp_\beta dr_\beta F^i_{\alpha,\beta}(r_\alpha - r_\beta, v_\alpha, v_\beta)\int_{-\infty}^{0} d\tau$$

$$\Big(e_\alpha\left\{E^j(r_\alpha + v_\alpha\tau, t+\tau) + \frac{1}{c}[v_\alpha, B(r_\alpha + v_\alpha\tau, t+\tau)]^j\right\}$$

$$\left\{\frac{\partial}{\partial p_\alpha^j} - \frac{\partial v_\alpha^\ell}{\partial p_\alpha^j}\tau\frac{\partial}{\partial r_\alpha^\ell}\right\} + e_\beta\left\{E^j(r_\beta + v_\beta\tau, t+\tau) + \frac{1}{c}[v_\beta, B(r_\beta + v_\beta\tau, t+\tau)]^j\right\}$$

$$\left.\left\{\frac{\partial}{\partial p_\alpha^j} - \frac{\partial v_\beta^\ell}{\partial p_\beta^j}\tau\frac{\partial}{\partial r_\beta^\ell}\right\}\right) g_{\alpha\beta}(r_\alpha + v_\alpha\tau, r_\beta + v_\beta\tau, p_\alpha, p_\beta, t-\tau) \qquad (2.10)$$

Here $g_{\alpha\beta}$ is determined by formula (2.6). As in the case of formula (2.8) it should be kept in mind that for sufficiently small values of $r_\alpha - r_\beta$ our treatment is inapplicable.

Another accounting for the correction due to the self-consistent field is based on the variation of the force acting on a particle α on the part of a particle β, which is associated with the nonuniformity of the motion in the self-consistent field. Here

$$\delta F_{\alpha,\beta}(r_\alpha, r_\beta, v_\alpha, v_\beta, t) = \frac{4\pi e_\alpha e_\beta c}{(2\pi)^3}\int_{-\infty}^{t} dt' \int dK e^{iK, r_\alpha - r_\beta - v_\beta(t'-t)}$$

$$\Big[- \frac{\sin cK(t-t')}{K} \left\{K - \frac{\left[v_\alpha [K v_\beta]\right]}{c^2}\right\} \int_t^{t'} dt''(K\delta v_\beta(t'')) -$$

$$- \frac{i}{c^2}\left[v_\alpha [K, \delta v_\beta(t')]\right] \quad - \frac{1}{c} \cos cK(t-t')\Big\{\delta v_\beta(t) -$$

$$- i v_\beta \int_t^{t'} dt''(K\delta v_\beta(t''))\Big\}\Big] , \tag{2.11}$$

where

$$\delta v_\alpha = \frac{1}{m_\alpha} \sqrt{1 - \frac{v_\alpha^2}{c^2}} \left\{\delta p_\alpha + \frac{1}{c^2} v_\alpha (v_\alpha \delta p_\alpha)\right\} \tag{2.12}$$

$$\delta p_\alpha = e_\alpha \int_{-\infty}^{t} dt' \left\{E(r_\alpha + v_\alpha(t'-t), t') + \frac{1}{c}[v_\alpha, B(r_\alpha + v_\alpha(t'-t), t')]\right\} \tag{2.13}$$

The corresponding correction for the correlation function is determined by a formula similar to formula (2.6):

$$\Delta g_{\alpha\beta}(r_\alpha, r_\beta, p_\alpha, p_\beta, t) = - \int_{-\infty}^{0} d\tau \Big\{\delta F_{\beta,\alpha}(r_\beta + v_\beta\tau, r_\alpha + v_\alpha\tau, v_\beta, v_\alpha, t+\tau)$$

$$f_\alpha(r_\alpha + v_\alpha\tau, p_\alpha, t+\tau)\left(\frac{\partial}{\partial p_\beta} - \frac{\partial v_\beta^i}{\partial r_\beta^i}\tau \frac{\partial}{\partial r_\beta^i}\right) f_\beta(r_\beta + v_\beta\tau, p_\beta, t+\tau) +$$

$$+ \delta F_{\alpha,\beta}(r_\alpha + v_\alpha\tau, r_\beta + v_\beta\tau, v_\alpha, v_\beta, t+\tau) f_\beta(r_\beta + v_\beta\tau, p_\beta, t+\tau)$$

$$\left(\frac{\partial}{\partial p_\alpha} - \frac{\partial v_\alpha^i}{\partial p_\alpha}\tau \frac{\partial}{\partial r_\alpha^i}\right) f_\alpha(r_\alpha + v_\alpha\tau, p_\alpha, t+\tau)\Big\} \tag{2.14}$$

As a result we obtain the following correction term to the right member of equation (2.7):

$$\Delta \mathfrak{I}_\alpha = \sum_\beta \int dp_\beta dr_\beta \frac{N_\beta}{V} \left\{ \frac{\partial g_{\alpha\beta}}{\partial p_\alpha} \delta F_{\alpha,\beta}(r_\alpha, r_\beta, v_\alpha, v_\beta t) + \right.$$

$$\left. + F_{\alpha,\beta}(r_\alpha - r_\beta, v_\alpha, v_\beta) \frac{\partial \Delta g_{\alpha\beta}}{\partial p_\alpha} \right\} \qquad (2.15)$$

In conclusion let us indicate that the kinetic equation obtained here may be utilized, for example, to obtain the high-frequency dielectric permeability of a relativistic plasma in an approximation with respect to e^2 higher than the usual approximation of self-consistent interaction.

§ 3. High-Frequency Dielectric Permeability of a Plasma.

(1) This part of our communication deals with the question of the dielectric permeability of a plasma in the region of high frequencies. Below we shall be primarily concerned with the region of frequencies in which the variable frequency ω considerably exceeds the Langmuir frequency of electrons $\omega_{Le} = \sqrt{4\pi Ne^2/m}$. On the other hand let us consider that the variable frequency is still much smaller than

$$\omega_{max} = \frac{(\varkappa T)^{\frac{3}{2}}(2m)^{-\frac{1}{2}}}{|e_i e|}$$

For the description of a plasma in these conditions the kinetic equation for rapidly variable processes obtained in § 1 is suitable. In such conditions for an isotropic plasma, as is known[4], there is an imaginary correction to the dielectric permeability which is quadratic with respect to the number of particles in a unit of volume, whose dependence on the frequency has the form $\omega^{-3} \ln(\omega/\omega_{max})$. Below the corresponding real correction is obtained, which depends on the frequency according to the law ω^{-3} sgn ω. Finally, a correction is also obtained to the tensor of dielectric permeability of the plasma in a strong magnetic field in conditions in which not only the variable frequency, but also the gyroscopic frequency of the electrons (and ions) is not small in comparison with the Langmuir frequency of the electrons.

(2) In order to obtain the dielectric permeability of a plasma, let us utilize the kinetic equation for steady-state rapidly variable processes. In the case of a plasma in a spatially homogeneous variable electric field E and a constant magnetic field B and for spatially homogeneous distributions, equation (1.6) may be written in the following form:

$$\frac{\partial f_\alpha}{\partial t} + e_\alpha (E + \frac{1}{c} [v_\alpha B]) \frac{\partial f_\alpha}{\partial p_\alpha} =$$

$$= \int_{-\infty}^{t} dt' \Big\{ \frac{\partial}{\partial p_\alpha^i} D_{ij}(p_\alpha, t', t) \frac{\partial f_\alpha (P_\alpha [t', t, p_\alpha], t')}{\partial P_\alpha^j [t', t, p_\alpha]} -$$

$$- \frac{\partial}{\partial p_\alpha^i} A_i(p_\alpha, t', t) f_\alpha(P_\alpha[t', t, p_\alpha], t') \Big\} \qquad (3.1)$$

here

$$D_{ij}(p_\alpha, t', t) = \sum_\beta \frac{N_\beta}{V} \int dp_\beta dr_\beta \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{dr_\alpha^i} f_\beta(P_\beta[t', t, p_\beta], t')$$

$$\frac{\partial}{\partial r_\alpha^i} U_{\alpha\beta}(|R_\alpha[t', t, p_\alpha, r_\alpha] - R_\beta[t', t, p_\beta, r_\beta]|) \ , \qquad (3.2)$$

$$A_i(p_\alpha, t', t) = \sum_\beta \frac{N_\beta}{V} \int dp_\beta dr_\beta \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{\partial r_\alpha^i} \frac{\partial f_\beta(P_\beta[t', t, p_\beta], t')}{\partial P_\beta^j[t', t, p_\beta]}$$

$$\frac{\partial}{\partial r_\alpha^j} U_{\alpha\beta}(|R_\alpha[t', t, p_\alpha, r_\alpha] - R_\beta[t', t, p_\beta, r_\beta]|) \ . \qquad (3.3)$$

The functions P and R are determined by formulas (1.4) and (1.5), and e_α , m_α , r_α , v_α , p_α are respectively the charge, mass, coordinates, velocity, and momentum of a particle of the α-type, $\Omega_\alpha = e_\alpha B/m_\alpha c$ is the gyroscopic frequency, N_α is the number of particles of the type α in a unit of volume; and, finally, $U_{\alpha\beta} = e_\alpha e_\beta/r$.

Equation (3.1) is obtained in the assumption of the weakness of interaction of the particles and therefore is inapplicable for small collision parameters. In this connection, in integration with respect to the impact parameters we shall have to break off at p_{min} . On the other hand, in equation (3.1) screening of the Coulomb interaction at great distances is not taken into account. The inapplicability of equation (3.1) to the investigation of collisions at large impact parameters may be manifested for sufficiently slow processes, when it is necessary to break off at p_{max}.*

(2) Let us investigate first the case of an isotropic plasma, when a constant magnetic field is absent. Here, without undertaking the task of accounting for the spatial dispersion of the dielectric permeability, let us consider the distributions of the particles as spatially homogeneous. Then for a slight deviation from the Maxwell distribution $f_\alpha^{(0)}$, linearizing the kinetic equation, and keeping in view that δf_α is proportional to the electric field, which is assumed to be weak, we obtain

$$\frac{\partial \delta f_\alpha}{\partial t} - \frac{e_\alpha}{\varkappa T} E v_\alpha f_\alpha^{(0)} = \sum_\beta N_\beta \frac{\partial}{\partial p_\alpha^i} \int dp_\beta dr_\beta \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{\partial r_\alpha^i}$$

$$\int_{-\infty}^{0} d\tau \left\{ \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta + (v_\alpha - v_\beta) r|)}{\partial r_\alpha^j} \left[\frac{\partial}{\partial p_\alpha^j} - \frac{\partial}{\partial p_\beta^j} \right] \left[f_\alpha^{(0)} \delta f_\beta (p_\beta, t+\tau) \right. \right.$$

$$\left. + \delta f_\alpha (p_\alpha, t+\tau) f_\beta^{(0)} \right] - \frac{f_\alpha^{(0)} f_\beta^{(0)}}{(\varkappa T)^2} U_{\alpha\beta}(|r_\alpha - r_\beta + (v_\alpha - v_\beta)\tau|)$$

$$\left. (e_\alpha v_\alpha + e_\beta v_\beta, E(t+\tau)) \right\} \qquad (3.4)$$

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* Let us note that in disregarding collisions the equality

$$f_\alpha(P_\alpha[t+\tau, t, p_\alpha], R_\alpha[t+\tau, t, p_\alpha, r_\alpha], t+\tau) = f_\alpha(p_\alpha, r_\alpha, t)$$

obtains, making it possible in the right member of equation (3.1) to transform the arguments of the functions.

In the solution of equation (3.4) let us consider the collision integral to be small. For the periodic dependence on time ($e^{-i\omega t}$) the fulfillment of the inequality $\omega >> \nu_{eff.}$ is necessary for this, where $\nu_{eff.}$ is determined below. Then in the first approximation

$$\delta f_{\alpha}^{(1)} = i \frac{e_{\alpha}}{\varkappa T} \frac{\vec{E}\vec{v}_{\alpha}}{\omega} f_{\alpha}^{(0)} \tag{3.5}$$

Substituting in the right member of equation (3.4) as δf_{α} the expression (3.5) we obtain in the second approximation

$$\delta f_{\alpha}^{(2)} = \frac{1}{\omega^2} \sum_{\beta} N_{\beta} \frac{\partial}{\partial p_{\alpha}^{i}} \int d\vec{p}_{\beta} d\vec{r}_{\beta} \left(\partial U_{\alpha}(\vec{r}_{\alpha} - \vec{r}_{\beta}) \Big/ \partial r_{\alpha}^{i} \right) \int_{-\infty}^{0} dr \quad e^{-i\omega t}$$

$$\frac{\partial U_{\alpha\beta}(|\vec{r}_{\alpha} - \vec{r}_{\beta} + (\vec{v}_{\alpha} - \vec{v}_{\beta})\tau|)}{\partial r_{\alpha}^{j}} \frac{f_{\alpha}^{(0)} f_{\beta}^{(0)}}{(\varkappa T)^2} \left(\frac{e_{\beta}}{m_{\beta}} - \frac{e_{\alpha}}{m_{\alpha}} \right) E_j \tag{3.6}$$

Formulas (3.5) and (3.6) make it possible to find the expression for the current density

$$j = \sum_{\alpha} e_{\alpha} N_{\alpha} \int dp_{\alpha} \mathbf{v}_{\alpha} \delta f_{\alpha} \quad , \tag{3.7}$$

and thereby to determine the value of the tensor of complex conductivity

$$\sigma_{ij} (j_i = \sigma_{ij} E_j)$$

or the tensor of complex dielectric permeability

$$\varepsilon_{ij} = \delta_{ij} + \frac{4\pi i \sigma_{ij}}{\omega}$$

For the isotropic plasma in question here these tensors are diagonal and in accordance with formulas (3.5)--(3.7):

$$\varepsilon(\omega) = 1 - \sum_{\alpha} \frac{4\pi e_\alpha^2 N_\alpha}{\omega^2 m_\alpha} + \frac{4\pi i}{\omega^2} \sum_{\alpha\beta} \frac{e_\alpha}{m_\alpha} \left(\frac{e_\alpha}{m_\alpha} - \frac{e_\beta}{m_\beta} \right)$$

$$\frac{N_\alpha N_\beta}{\varkappa T} \int dp_\alpha dp_\beta f_\alpha^{(0)} f_\beta^{(0)} \int_{-\infty}^{0} d\tau e^{-i\omega\tau} \int_{K_{min}}^{K_{max}} \frac{dK}{(2\pi)^3}$$

$$\frac{4\pi e_\alpha e_\beta}{3K^2} \quad e^{i(K, v_\alpha - v_\beta)\tau} , \qquad (3.8)$$

where

$$K_{max} = \rho_{min}^{-1} = \frac{\varkappa T}{|e_\alpha e_\beta|} \quad , \quad K_{min} = \rho_{max}^{-1} \approx r_D^{-1}$$

(r_D is the Debye radius).

If in the right member of formula (3.8) we disregard the terms containing positive degrees of the ratio of the mass of the electron and the mass of the ion, then in the assumption that there is only one type of ions, we obtain:

$$\varepsilon(\omega) = 1 - \frac{\omega_{Le}^2}{\omega^2} + i \frac{\omega_{Le}^2}{\omega^3} \frac{4}{3} \frac{\sqrt{2\pi}\ (ee_i)^2 N_i}{\sqrt{m}\ (\varkappa T)^{\frac{3}{2}}} F(\omega) , \qquad (3.9)$$

where

$$F(\omega) = \int_0^\infty \frac{d\tau}{\tau} e^{i\omega\tau}\left[\mathrm{Erf}\left(\tau\sqrt{\frac{\varkappa T}{2m}}\, K_{max}\right) - \mathrm{Erf}\left(\tau\sqrt{\frac{\varkappa T}{2m}}\, K_{min}\right)\right] \qquad (3.10)$$

Here

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is the integral of probability. Keeping in view that $K_{max} >> K_{min}$, and also the fact that these quantities themselves are determined with an accuracy to a factor of an order of unity, formula (3.10) may be rewritten in the form:

$$F(\omega) \equiv F'(\omega) + iF''(\omega) = \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{\tau} \cos\omega\tau + i \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{\tau} \sin \omega\tau \qquad (3.11)$$

where

$$\tau_{min} = \sqrt{\frac{2m}{\varkappa T}}\, K_{max}^{-1}\ , \qquad \tau_{max} = \sqrt{\frac{2m}{\varkappa T}}\, K_{min}^{-1} \approx \omega^{-1}$$

In conditions in which $\omega << \omega_{Le}$

$$F'(\omega) = \ln \frac{K_{max}}{K_{min}}\ , \qquad F''(\omega) \cong \frac{\omega}{\omega_{Le}} \qquad (3.12)$$

The latter expression for F'' leads to a small correction of the term $\varepsilon(\omega)$ proportional to ω^{-2}. Therefore we may speak of a change of ω^2_{Le} by the quantity $\Delta\omega^2_{Le}$ where

$$\Delta \approx \frac{4}{3} \frac{\sqrt{2\pi}\,(ee_i)^2 N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}} \frac{1}{\omega_{Le}}$$

Expression (3.10) for $F'(\omega)$ leads to the usual effective frequency of collisions, making it possible to write for the dielectric permeability the following expression[4,9,10].

$$\varepsilon(\omega) = 1 - \frac{\omega^2_{Le}}{\omega^2}(1 + \Delta) + i\,\frac{\omega^2_{Le}}{\omega^3}\,\nu^{(0)}_{eff.} \qquad (3.13)$$

where

$$\nu^{(0)}_{eff.} = \frac{4}{3} \frac{\sqrt{2\pi}\,(ee_i)^2 N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}} \ln\left(\frac{\varkappa T}{|ee_i|}\, r_D\right) \qquad (3.14)$$

For frequencies considerably exceeding the Langmuir frequency of the electron ($\omega \gg \omega_{Le}$), we have from (3.11) and

$$F'(\omega) = \ln\left(\frac{K_{max}}{\gamma|\omega|}\sqrt{\frac{\varkappa T}{2m}}\right), \quad F''(\omega) = \frac{\pi}{2}\,\mathrm{sgn}\,\omega \qquad (3.15)$$

$\gamma = 1.781$ is Euler's constant. Substituting expression (3.15) in formula (3.9) we obtain

$$\varepsilon(\omega) = \varepsilon' + i\varepsilon'' = 1 - \frac{\omega_{ie}^2}{\omega^2} + i\,\frac{\omega_{Le}^2}{\omega^3}\,\nu_{eff.}^{(\omega)} - \frac{\omega_{Le}^2}{\omega^3}\,\omega_{eff}\,\mathrm{sgn}\,\omega\,, \tag{3.16}$$

where

$$\omega_{eff.} = \frac{(2\pi)^{\frac{3}{2}}\,(ee_i)^2 N_i}{3\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}}\,, \tag{3.17}$$

and $\nu_{eff.}^{(\omega)}$ has the form familiar from the theory of the absorption of radio waves in interplanetary gas[4], and equalling

$$\nu_{eff.}^{(\omega)} = \frac{4}{3}\,\frac{\sqrt{2\pi}\,(ee_i)^2 N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}}\,\ln\left|\frac{(\varkappa T)^{\frac{3}{2}}}{\gamma\omega\sqrt{2m}\,|ee_i|}\right| \tag{3.18}$$

The quantity $\nu_{eff.}$ differs in order of magnitude from $\omega_{eff.}$ by a large logarithm. Inasmuch as this logarithm is determined with a certain inaccuracy associated with the choice of K_{max}, the question may arise of the expediency of retaining a curve proportional to $\omega_{eff.}$. However, in actuality, by virtue of the fact that $\nu_{eff.}$ makes a contribution to the imaginary part of the dielectric permeability, and $\omega_{eff.}$, to the real part, the retention of a member proportional to $\omega_{eff.}$ is not in excess of accuracy.

It is necessary to note an important difference in the correction to the real part $\varepsilon(\omega)$ in the region $\omega \ll \omega_{Le}$ and in the region $\omega \gg \omega_{Le}$. Although in the second case the absolute value of the correction is smaller than the value of the correction in the first (low-frequency) case, nevertheless in the region $\omega \gg \omega_{Le}$ a new dependence on the frequency arises, which in principle makes it possible to find the corresponding correction. The complexity of finding it is associated with the necessity for the fulfillment of the condition

$$\omega \ll \frac{(\varkappa T)^{\frac{3}{2}} m^{-\frac{1}{2}}}{|ee_i|}$$

Here, for example, in the expression for the refractive index

$$n = \sqrt{\varepsilon'} \cong 1 - \frac{\omega_{Le}^2}{2\omega^2} - \frac{1}{8}\frac{\omega_{Le}^4}{\omega^4} - \frac{1}{2}\frac{\omega_{Le}^2 \omega_{eff.}}{\omega^3} - \frac{1}{16}\frac{\omega_{Le}^6}{\omega^6} \qquad (3.19)$$

the third term $(\sim\omega^{-4})$ will always be larger than the fourth, proportional to ω^{-3}. Therefore the correction obtained may manifest itself only with the determination of the refractive index as a function of the frequency with extremely great accuracy. Let us indicate that the fourth term of the right member of formula (3.19) is, on one hand, smaller than the third, and on the other hand, larger than the fifth. For the condition

$$N_e^{\frac{1}{3}} (\varkappa T)^{\frac{1}{2}} m^{-\frac{1}{2}} \ll \omega \ll \frac{(\varkappa T)^{\frac{3}{2}} m^{-\frac{1}{2}}}{|ee_i|}$$

This region of frequencies is rather broad, for it is determined by the inequality

$$|ee_i| N_e^{\frac{1}{3}} << \varkappa T.$$

(4) Let us turn now to the investigation of a plasma placed in a constant magnetic field. In this case the linearized kinetic equation for a slight deviation from the Maxwell distribution may be written in the following form

$$\frac{\partial \delta f_\alpha}{\partial t} + \frac{e_\alpha}{c} [v_\alpha B] \frac{\partial \delta f_\alpha}{\partial p_\alpha} - \frac{e_\alpha}{\varkappa T} E v_\alpha f_\alpha^{(0)} = \sum_\beta N_\beta \frac{\partial}{\partial p_\alpha^i} \int dp_\beta dr_\beta$$

$$\frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{dr_\alpha^i} \int_{-\infty}^{0} d\tau \Big\{ \Big[\frac{\partial}{\partial r_\alpha^i} U_{\alpha\beta}\Big(|R_\alpha^o(\tau, p_\alpha, r_\alpha) - R_\beta^o(\tau, p_\beta, r_\beta)| \Big) \Big]$$

$$\Big[\frac{\partial}{\partial P_\alpha^{oj}(\tau, p_\alpha)} - \frac{\partial}{\partial P_\beta^{oj}(\tau, p_\beta)} \Big] \Big[f_\alpha^{(0)} \delta f_\beta (P_\beta^o(\tau, p_\beta), t+\tau) +$$

$$+ \delta f_\alpha (P_\alpha^o(\tau, p_\alpha), t+\tau) f_\beta^{(0)} \Big] + \frac{f_\alpha^{(0)} f_\beta^{(0)}}{(\varkappa T)^2} \times$$

$$\times U_{\alpha\beta}(|R_\alpha^o(\tau, p_\alpha, r_\alpha) - R_\beta^o(\tau, p_\beta, r_\beta)|) \times$$

$$\times \Big(E(t+\tau), \frac{e_\alpha}{m_\alpha} P_\alpha^o(\tau, p_\alpha) + \frac{e_\beta}{m_\beta} P_\beta^o(\tau, p_\beta) \Big) \Big\} \qquad (3.20)$$

Here P^o and R^o are determined by formulas (1.4) and (1.5), if in the latter the electric field is assumed equal to zero.

For a field depending on the time periodically $e^{-i\omega t}$ equation (3.20) may be solved according to the theory of perturbations, assuming the right member of this equation to be small, if the condition

$$|\omega^2 \pm \Omega_\alpha^2| >> \nu_{eff}^2.$$

is fulfilled. Below this condition will be considered fulfilled. Then in an approximation disregarding the collision integral, we obtain

$$\delta f_\alpha^{(1)}(p_\alpha, t) = \frac{e_\alpha f_\alpha^{(0)}}{m_\alpha \varkappa T} A_{sr}(\omega, \Omega_\alpha) p_\alpha^r E_s e^{-i\omega t} , \qquad (3.21)$$

where

$$A_{sr} = \left\{ \frac{B_s B_r}{B^2} \frac{i}{\omega} - \frac{\Omega_\alpha}{\omega^2 - \Omega_\alpha^2} \varepsilon_{s\ell r} \frac{B_e}{B} - \frac{i\omega}{\omega^2 - \Omega_\alpha^2} \frac{B_s B_r - B^2 \delta_{sr}}{B^2} \right\} \qquad (3.22)$$

Here $\varepsilon_{s\ell r}$ is a completely antisymmetric tensor. Utilizing equation (3.21) we obtain the following equation for the second approximation to the nonequilibrium part of the distribution function:

$$\frac{\partial \delta f_\alpha^{(2)}}{\partial t} + \frac{e_\alpha}{c} [v_\alpha B] \frac{\partial \delta f_\alpha^{(2)}}{\partial p_\alpha} = \sum_\beta N_\beta \frac{\partial}{\partial p_\alpha^i} \int dp_\beta \frac{f_\beta^{(0)} f_\alpha^{(0)}}{\varkappa T} \times$$

$$\times \int dr_\beta \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{\partial r_\alpha^i} \int_{-\infty}^{0} d\tau e^{-i\omega\tau} \times$$

$$\times \left\{\frac{\partial}{\partial r_\alpha^j} U_{\alpha\beta}\left(|R_\alpha^o(\tau, p_\alpha, r_\alpha) - R_\beta^o(\tau, p_\beta, r_\beta)|\right)\right\} \times$$

$$\times \left[\frac{e_\alpha}{m_\alpha} A_{sj}(\omega, \Omega_\alpha) - \frac{e_\beta}{m_\beta} A_{sj}(\omega, \Omega_\beta)\right] e^{-i2t} E_s \qquad (3.23)$$

In the case of a steady-state periodic process, to be examined by us, the solution of this equation may be written in the form:

$$\delta f_\alpha^{(2)}(p_\alpha, t) = \int_{-\infty}^{t} dt' \sum_\beta N_\beta \frac{\partial}{\partial P_\alpha^{oi}(t'-t, p_\alpha)} \int dp_\beta \frac{f_\beta^{(0)} f_\alpha^{(0)}}{\varkappa T}$$

$$\int dr_\beta \frac{\partial U_{\alpha\beta}(|r_\alpha - r_\beta|)}{dr_\alpha^i} \int_{-\infty}^{0} d\tau e^{-i\omega\tau}$$

$$\left\{\frac{\partial}{\partial r_\alpha^j} U_{\alpha\beta}\left(|R_\alpha^o(\tau, p_\alpha^o[t'-t], r_\alpha) - R_\beta^o(\tau, p_\beta, r_\beta)|\right)\right\} \times$$

$$\times \left[\frac{e_\alpha}{m_\alpha} A_{sj}(\omega, \Omega_\alpha) - \frac{e_\beta}{m_\beta} A_{sj}(\omega, \Omega_\beta)\right] e^{-i\omega t'} E_s \qquad (3.24)$$

In order to determine the tensor of complex dielectric permeability let us substitute expressions (3.21) and (3.24) in formula (3.7) and integrate with respect to the momenta and the time (t'). Here, in particular, let us take into account that

$$\int_0^\infty dt e^{i\omega t} \frac{\partial}{\partial p_\alpha^i} P_\alpha^r(t, p_\alpha) = A_{ri}(\omega, -\Omega_\alpha) \qquad (3.25)$$

Then we obtain

$$\varepsilon_{ij}(\omega) = \delta_{ij} + \sum_{\alpha} \frac{4\pi e_{\alpha}^{2} N_{\alpha}}{\omega m_{\alpha}} iA_{ji}(\omega,\Omega_{\alpha}) - \frac{4\pi i}{\omega} \sum_{\alpha\beta} \frac{e_{\alpha}}{m_{\alpha}} \times$$

$$\times A_{ir}(\omega,-\Omega_{\alpha})\left[\frac{e_{\alpha}}{m_{\alpha}} A_{js}(\omega,\Omega_{\alpha}) - \frac{e_{\beta}}{m_{\beta}} A_{js}(\omega,\Omega_{\beta})\right] \times$$

$$\times \frac{N_{\alpha}N_{\beta}}{\varkappa T} (4\pi e_{\alpha}e_{\beta})^{2} \int_{-\infty}^{0} d\tau e^{i\omega\tau} \int_{K_{min}}^{K_{max}} \frac{dK}{(2\pi)^{3}} \frac{K_{r}K_{s}}{K^{4}}$$

$$\exp\left\{- \frac{\varkappa T}{2}\left[\frac{1}{m_{\alpha}} + \frac{1}{m_{\beta}}\right]\left(\frac{KB}{B}\right)^{2}\tau^{2} - 2\varkappa T \frac{B^{2}K^{2}-(BK)^{2}}{B^{2}} \left[\frac{\sin^{2}(\frac{\Omega_{\alpha}\tau}{2})}{m_{\alpha}\Omega_{\alpha}^{2}} +\right.\right.$$

$$\left.\left. + \frac{\sin^{2}(\frac{\Omega_{\beta}\tau}{2})}{m_{\beta}\Omega_{\beta}^{2}}\right]\right\} \qquad (3.26)$$

Assuming that in a plasma there are electrons and only one type of ions and disregarding corrections of the order of the powers of the ratio of the mass of the electron to the mass of the ion we obtain:

$$\varepsilon_{ij}(\omega) = \varepsilon_{ij}^{(0,h)} + \delta\varepsilon_{ij}^{(a)} + \delta\varepsilon_{ij}^{(h)} \qquad (3.27)$$

where $\varepsilon_{ij}^{(0,h)}$ is the hermitian part of the tensor of dielectric permeability obtained with complete disregard of the collision integral;

$$\varepsilon_{ij}^{(0h)} = \delta_{ij} - \frac{\omega_{ie}^{2}}{\omega^{2}} \left\{\frac{B_{i}B_{j}}{B^{2}} - \frac{\omega^{2}}{\Omega_{e}}\left[\frac{\Omega_{e}}{\omega^{2}-\Omega^{2}} - \frac{\Omega_{i}}{\omega^{2}-\Omega_{i}^{2}}\right] \frac{B_{i}B_{j}-\delta_{ij}B^{2}}{B^{2}} +\right.$$

$$+ \frac{i\omega}{\Omega_e} \left[\frac{\Omega_e^2}{\omega^2-\Omega_e^2} - \frac{\Omega_i^2}{\omega^2-\Omega_i^2}\right] \varepsilon_{j\ell i} \frac{B_\ell}{B}\Big\} \tag{3.28}$$

and $\delta\varepsilon_{ij}^{(h)}$ and $\delta\varepsilon_{ij}^{(a)}$ are, respectively, the hermitian and antihermitian parts of the tensor of dielectric permeability arising with the taking into account of the collision integral:

$$\delta\varepsilon_{ij} = \delta\varepsilon_{ij}^{(h)} + \delta\varepsilon_{ij}^{(a)} = i\,\frac{\omega_{Le}^2}{\omega^3}\,\omega_{eff.}\left\{\frac{B_iB_j}{B^2} F_1(\omega) + [F_1(\omega)+F_2(\omega)]T_{ij}^1\right\} \tag{3.29}$$

where

$$T_{ij}^1 = -\frac{2i\omega^3}{\Omega_e^2}\varepsilon_{ji\ell}\frac{B_\ell}{B}\left[\frac{\Omega_e}{\omega^2-\Omega_e^2} - \frac{\Omega_i}{\omega^2-\Omega_i^2}\right]\left[\frac{\Omega_e^2}{\omega^2-\Omega_e^2} - \frac{\Omega_i^2}{\omega^2-\Omega_i^2}\right] -$$

$$\frac{\omega^2}{\Omega_e^2}\,\frac{B_iB_j-\delta_{ij}B^2}{B^2}\left\{\omega^2\left[\frac{\Omega_e}{\omega^2-\Omega_e^2} \quad \frac{\Omega_i}{\omega^2-\Omega_i^2}\right]^2 + \left[\frac{\Omega_e^2}{\omega^2-\Omega_e^2} - \frac{\Omega_i^2}{\omega^2-\Omega_i^2}\right]^2\right\}$$

The functions F_1 and F_2 are determined by formulas:

$$F_1(\omega) = \frac{3}{\pi}\int_0^\infty d\tau e^{i\omega\tau}\int_{-1}^{+1}\frac{dx x^2}{\sqrt{\varphi(x,\tau)}}\left\{\mathrm{Erf}\left(K_{max}\sqrt{\frac{\varkappa T}{2m}}\sqrt{\varphi(x,\tau)}\right) -\right.$$

$$\left.-\mathrm{Erf}\left(K_{min}\sqrt{\frac{\varkappa T}{2m}}\sqrt{\varphi(x,\tau)}\right)\right\}, \tag{3.30}$$

$$F_2(\omega) = \frac{3}{2\pi}\int_0^\infty d\tau 2^{i\omega\tau}\int_{-1}^{+1}\frac{dx(1-3x^2)}{\sqrt{\varphi(x,\tau)}}\left\{\mathrm{Erf}\left(K_{max}\sqrt{\frac{\varkappa T}{2m}}\sqrt{\varphi(x,\tau)}\right) -\right.$$

$$\left.-\mathrm{Erf}\left(K_{min}\sqrt{\frac{\varkappa T}{2m}}\sqrt{\varphi(x,\tau)}\right)\right\} \tag{3.31}$$

where

$$\varphi(x,\tau) = \left(1-\frac{m}{m_i}\right)x^2\tau^2 + 4(1-x^2)\left[\frac{\sin^2\left(\frac{\Omega_e\tau}{2}\right)}{\Omega_e^2} + \frac{m}{m_i}\,\frac{\sin^2\left(\frac{\Omega_i\tau}{2}\right)}{\Omega_i^2}\right] \tag{3.32}$$

Here the real parts of the functions F_1 and F_2 make a contribution to the antihermitian part of the tensor of dielectric permeability, and the imaginary parts, to the hermitian part.

For frequencies much larger than the electron gyroscopic frequency $(\omega \gg |\Omega_e|)$:

$$F_1(\omega) = \frac{2}{\pi} F(\omega) \qquad \text{and,} \qquad F_2(\omega) \approx 0 \tag{3.33}$$

Therefore in the region of these frequencies, and also $\omega \gg \omega_{ie}$ we have

$$\delta\varepsilon_{ij} = \frac{\omega_{ie}^2}{\omega^3} \left[i\, \nu^{(\omega)}_{eff.} - \omega_{eff.} \right] \left\{ \frac{B_i B_j}{B^2} + T^1_{ij} \right\} \tag{3.34}$$

If it should be found that ω and $\Omega_e \ll \omega_{Le}$ then $\delta\varepsilon_{ij}$ would have a similar form, while $\nu^{(0)}_{eff.}$ should be substituted in the place of $\nu^{(\omega)}_{eff.}$, and the quantity $\Delta\omega_{Le}$, in the place of $\omega_{eff.}$. Below let us consider the variable frequency to be greater than the Langmuir electron frequency. This, in particular, makes it possible in formulas (3.30) and (3.31) to assume $K_{min} = 0$. Let us investigate first the hermitian part of the correction to the tensor of dielectric permeability, for which we investigate the imaginary part of the functions F_1 and F_2. Let us note that inasmuch as the integrand of the corresponding imaginary parts do not have singularities for small values τ, in the formulas for them one may assume K_{max} equal to infinity. Therefore, considering $\Omega_c < 0$, we have:

$$F_1''(\omega) = \frac{3}{\pi}\int_0^\infty \frac{d\xi}{\xi}\sin\left(\xi\frac{2\omega}{|\Omega_\ell|}\right)\left\{\frac{1}{1-\psi(\xi)} - \frac{\psi(\xi)}{[1-\psi(\xi)]^{\frac{3}{2}}}\ \ell n\ \frac{1+\sqrt{1-\psi(\xi)}}{\sqrt{\psi(\xi)}}\right\} \tag{3.35}$$

$$F_2''(\omega) = \frac{3}{\pi}\int_0^\infty \frac{d\xi}{\xi}\frac{\sin\left(\frac{\xi 2\omega}{|\Omega_\ell|}\right)}{\sqrt{1-\psi(\xi)}}\left\{-\frac{3}{2}\frac{1}{\sqrt{1-\psi(\xi)}} + \left[1+\frac{3}{2}\frac{\psi(\xi)}{1-\psi(\xi)}\right]\ell n\ \frac{1+\sqrt{1-\psi(\xi)}}{\sqrt{\psi(\xi)}}\right\}, \tag{3.36}$$

where

$$\psi(\xi) = \frac{\sin^2\xi + \frac{e^2}{e_i^2}\frac{m_i}{m}\sin^2\left(\frac{e_i}{e}\frac{m}{m_i}\xi\right)}{\left[1+\left(\frac{m}{m_i}\right)\right]\xi^2} \tag{3.37}$$

In order to obtain relatively simple formulas let us analyze some extreme cases, considering still, however, that the gyroscopic frequency of the electron and the variable frequency are small in comparison with

$$\omega_{max} = \frac{(\varkappa T)^{\frac{3}{2}} m^{-\frac{1}{2}}}{|ee_i|}$$

The case of the largest frequencies ω, when the magnetic field has no effect on the collisions, corresponds to formulas (3.33) and (3.34). Therefore let us assume further that

$|\Omega_e| \gg \omega$. With the fulfillment of this inequality the main contribution in integrals (3.35) and (3.36) is made by the region $\xi \gg 1$, in which $\Psi(\xi)$ is small in comparison with unity. Three regions of large values of ξ should be distinguished. These, firstly, are the region

$$1 \ll \xi \ll \sqrt{\frac{m_i}{m}}$$

in which

$$\Psi(\xi) \cong \frac{\sin^2\xi}{\xi^2} \qquad (3.38)$$

Secondly, the region determined by the inequality

$$\sqrt{\frac{m_i}{m}} < \xi < \left|\frac{em_i}{e_i m}\right|$$

in which

$$\Psi(\xi) \simeq \frac{m}{m_i} \qquad (3.39)$$

Finally, $\xi > \left|\frac{em_i}{e_i m}\right|$, for which

$$(\xi) \cong \frac{1}{\xi^2}\frac{e^2}{e_i^2}\frac{m_i}{m}\sin^2\left(\frac{e_i}{e}\frac{m}{m_i}\xi\right) \qquad (3.40)$$

Keeping in view the smallness of $\Psi(\xi)$ we obtain immediately from formula (3.35):

$$F_1''(\omega) = \frac{3}{2}\ \mathrm{sgn}\ \omega \tag{3.41}$$

The matter is somewhat more complicated in relation to the function $F_2''(\omega)$. Here, in accordance with the three regions of the values of ξ and the corresponding values of the functions $\Psi(\xi)$, we have:

$$F_2''(\omega) = \frac{3}{2}\ ln\ \left|\frac{\Omega_e}{\omega}\right| \mathrm{sgn}\ \omega \quad , \quad |\Omega_e|\sqrt{\frac{m}{m_i}} << \omega << |\Omega_e| \tag{3.42}$$

$$F_2''(\omega) = \frac{3}{4}\left[ln\ \frac{4m_i}{m} - 3\right] \mathrm{sgn}\ \omega\ , \quad \Omega_i << \omega << |\Omega_e|\sqrt{\frac{m}{m_i}} \tag{3.43}$$

$$F_2''(\omega) = \frac{3}{2}\ ln\left(\frac{\Omega_i}{\omega}\sqrt{\frac{m_i}{m}}\right) \mathrm{sgn}\ \omega\ , \quad \omega << \Omega_i \tag{3.44}$$

Formulas (?.29) and (3.41)--(3.44) make it possible to write the hermitian part of the correction to the tensor of complex dielectric permeability in the following form:

$$\delta\varepsilon_{ij}^{(h)}(\omega) = -\frac{3}{2}\ \frac{\omega_{Le}^2 \omega_{eff.}}{\omega^3}\ \mathrm{sgn}\omega\left\{\frac{B_iB_j}{B^2} + ln\ \left|\frac{\Omega_e}{\omega}\right| T_{ij}^1\right\},$$

$$|\Omega_e|\sqrt{\frac{m}{m_i}} << \omega << |\Omega_e|\ , \tag{3.45}$$

$$\delta\varepsilon_{ij}^{(h)}(\omega) = -\frac{3}{2}\ \frac{\omega_{Le}^2 \omega_{eff.}}{\omega^3}\ \mathrm{sgn}\omega\left\{\frac{B_iB_j}{B^2} + T_{ij}^1\ \frac{1}{2}\left[ln\ \frac{4m_i}{m} - 1\right]\right\},$$

$$\Omega_i << \omega << |\Omega_e|\sqrt{\frac{m}{m_i}}\ , \tag{3.46}$$

$$\delta\varepsilon^{(h)}_{ij}(\omega) = -\frac{3}{2}\frac{\omega^2_{Le}\omega_{eff.}}{\omega^3}\,\mathrm{sgn}\,\omega\left\{\frac{B_iB_j}{B^2} + T^1_{ij}\,\ln\left|\frac{\Omega_i}{\omega}\sqrt{\frac{m_i}{m}}\right|\right\},$$

$$\omega \ll \Omega_i \qquad (3.47)$$

Let us proceed now to an investigation of the antihermitian correction. First of all let us note that, as in the investigation of the imaginary part of the function F_1 and F_2, in the real part of the function F_2 we may assume $K_{max} = \infty$. Then $F_2(\omega)$ assumes a form analogous to expression (3.36) only with the difference that a cosine appears instead of a sine. Therefore for $\omega \ll \Omega_e$ we have:

$$F'_2(\omega) = \frac{3}{2\pi}\int_1^\infty \frac{d\xi}{\xi}\cos\left(\xi\frac{2\omega}{\Omega_e}\right)\left\{\ln\frac{4}{\Psi(\xi)} - 3\right\} \qquad (3.48)$$

In accordance with formulas (3.38)--(3.40) we obtain from formula (3.48):

$$F'_2(\omega) = \frac{3}{2\pi}\left[\ln\frac{\omega}{\Omega_e}\right]^2, \qquad |\Omega_e|\sqrt{\frac{m}{m_i}} \ll \omega \ll |\Omega_e|, \qquad (3.49)$$

$$F'_2(\omega) = \frac{3}{2\pi}\left\{\ln\left|\frac{\Omega_e}{2\gamma\omega}\right|\left(\ln\frac{4m_i}{m} - 3\right) - \frac{1}{4}\left(\ln\frac{m_i}{m}\right)^2\right\},$$

$$\Omega_i \ll \omega \ll |\Omega_e|\sqrt{\frac{m}{m_i}} \qquad (3.50)$$

$$F'_2(\omega) = \frac{3}{2\pi}\left\{\left(\ln\frac{\Omega_i}{\omega}\right)^2 + \ln\left|\frac{\Omega_i}{\omega}\right|\ln\frac{m_i}{m} + \frac{3}{4}\left(\ln\frac{m_i}{m}\right)^2\right\},$$

$$\omega \ll \Omega_i \qquad (3.51)$$

In the expression for $F_1'(\omega)$ one cannot assume $K_{max} = \infty$. However, keeping in view the fact that for $\tau \ll 1/\Omega_e$ the integrand of formula (3.30) does not depend on the magnetic field, we may write the following formula for $F_1''(\omega)$, suitable both in the case $\omega \gg \Omega_e$

$$F_1'(\omega) = \frac{2}{\pi}\int_{\xi_{min}}^{1} \frac{d\xi}{\xi} \cos\left(\frac{2\omega}{\Omega_\ell}\xi\right) + \frac{3}{\pi}\int_{1}^{\infty} \frac{d\xi}{\xi}\cos\left(\frac{2\omega}{\xi^{-1}\Omega_\ell}\right) \qquad (3.52)$$

where

$$\xi_{min} = \frac{\sqrt{2m}\,|\Omega_\ell e e_i|}{(\varkappa T)^{\frac{3}{2}}}$$

In particular, for the case with which we are now concerned, we have:

$$F_1'(\omega) = \frac{2}{\pi}\ln\frac{(\varkappa T)^{\frac{3}{2}}}{\sqrt{2m}\,|\Omega_\ell e e_i|} + \frac{3}{\pi}\ln\left|\frac{\Omega_\ell}{2\omega}\right| \qquad (3.53)$$

Now by means of formulas (3.29), (3.49)--(3.51), and (3.53) we may write the following expression for the antihermitian part of tensor of complex dielectric permeability

$$\delta\varepsilon_{ij}^{(a)}(\omega) = i\left\{\frac{B_iB_j}{B^2}\left[\nu_{eff.}^{(\Omega)} + \delta\nu_{||}(\omega)\right] + \right.$$

$$\left. + T_{ij}^{\perp}\left[\nu_{eff.}^{(\Omega)} + \delta\nu_{\perp}(\omega)\right]\right\}\frac{\omega_{Le}^2}{\omega^3}, \qquad (3.54)$$

where

$$\nu^{(\Omega)}_{\text{eff.}} = \frac{4}{3} \frac{\sqrt{2\pi}\,(ee_i)^2 N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}} \ln \frac{(\varkappa T)^{\frac{3}{2}}}{\gamma \sqrt{2m}\,|\Omega_e ee_i|}, \tag{3.55}$$

$$\delta\nu_{\parallel}(\omega) = 2 \frac{\sqrt{2\pi}\,(ee_i)^2 N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}} \ln \left|\frac{\Omega_e}{2\omega}\right|, \tag{3.56}$$

$$\delta\nu_{\perp}(\omega) = \frac{\sqrt{2\pi}\,(ee_i) N_i}{\sqrt{m}\,(\varkappa T)^{\frac{3}{2}}} \begin{cases} [\ln \frac{\Omega_e}{\omega}]^2, \quad |\Omega_e|\sqrt{\frac{m}{m_i}} \ll \omega \ll |\Omega_e|, & (3.57) \\ [\ln \frac{4m_i}{m} - 1] \ln\left|\frac{\Omega_e}{\omega}\right| - \frac{1}{4}[\ln \frac{m_i}{m}]^2, & \\ \Omega_i \ll \omega \ll |\Omega_e| \sqrt{\frac{m}{m_i}}, & (3.58) \\ [\frac{\Omega_i}{\omega}]^2 + \ln \frac{m_i}{m} \ln\left|\frac{\Omega_i}{\omega}\right| + \frac{3}{4}[\ln \frac{m_i}{m}]^2, & \\ \omega \ll \Omega_i & (3.59) \end{cases}$$

A comparison of formulas (3.54)--(3.59) with formula (3.34) makes it possible to say that in the case of strong fields we may speak of two effective frequencies of collisions, or what is the same thing, of two times of relaxation.*

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* Formula (20) of study[11] may be regarded as an interpolation formula, giving in extreme cases formulas (3.18) and (3.55) of the present study. For $\omega \sim \Omega_e$ the approximate kernel of the collision integral, corresponding to formula (17) of study[1] gives a poor approximation.

Formula (3.57), if in it $\omega \sim \omega_{Le}$ is assumed and the leading term is retained, together with formula (3.55) leads to an expression for the transverse time of relaxation determining the coefficient of electron-ion diffusion across the magnetic field[11].

In all the formulas of our article it was assumed that the maximum frequency (ω_{max}) is determined by the inapplicability of the theory of perturbations. If, however, this is not the case, and the inapplicability of our formulas in the region of small impact parameters of the collisions will be determined by quantum mechanic effects disregarded in obtaining the kinetic equation (1), then ($\varkappa T/h$) should be taken as ω_{max}.

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